

MECHANICAL ENGINEERING

STRENGTH OF MATERIALS

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1.(a) If a round bar of 37.5 mm diameter and 2.4 m length is stretched by 2.5 mm, find its bulk modulus and lateral contraction. Take Young's modulus= 110 GN/m² and shear modulus = 42 GN/m² for the material of the bar. (5)

D= 37.5 mm = 37.5 x 10^{-3} m, l = 2.4 m, δ l= 2.5 mm = 2.5 x 10^{-3} m

 $E{=}\;110\;GN/m^2{=}\;110\;x\;10^9\;N/m^2$

 $G=42 \ GN/m^2=42 \ x \ 10^9 \ N/m^2$

Using relation, $E = 2G (1 + \frac{1}{m})$

$$110 \ge 10^9 = 2 \ge 42 \ge 10^9 (1 + 10^9)$$

$$(1 + \frac{1}{m}) = \frac{110 \times 10^9}{42 \times 10^9}$$
$$\cdot \frac{1}{m} = 0.30952$$

Bulk Modulus: $E = 3K \left(1 - \frac{2}{m}\right)$

 $\mathbf{K} = \frac{E}{3(1-\frac{2}{m})}$

 \therefore K = 0.96 x 10⁵ N/mm²

Longitudinal Strain: $e = \frac{\delta l}{l} = 1.0416 \text{ x } 10^{-3}$

Lateral contraction: $\frac{1}{m} = \frac{Lateral strain}{Longitudnal strain}$

$$\therefore 0.3095 = \frac{Lateral strain}{1.0413 \times 10^{-3}}$$

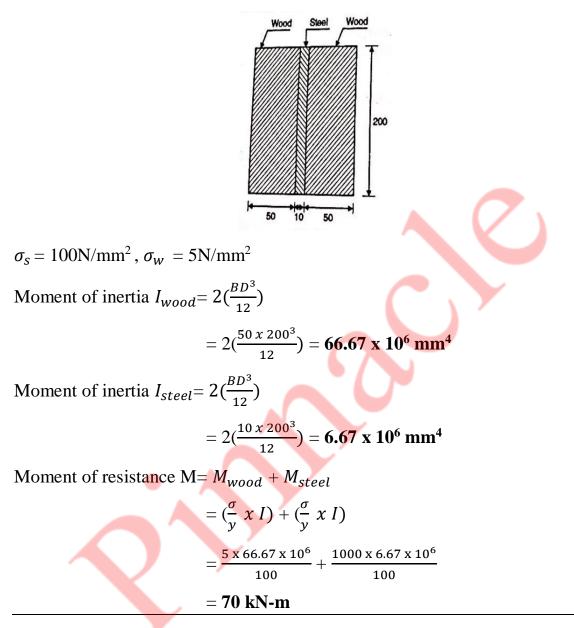
 \therefore Lateral strain = 3.223 x 10⁻⁴

Now, Lateral strain = $\frac{Lateral \ contraction}{original \ lateral \ dimension}$

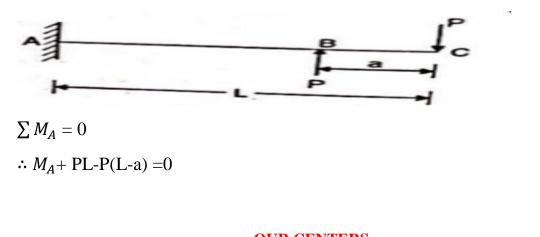
 \therefore lateral contraction = 1.208 x 10⁻⁵ m



1.(b)A flitched beam consists of steel and timber as shown in figure. Determine the moment of resistance of the beam. Take $\sigma_s = 100$ N/mm² and $\sigma_w = 5$ N/mm². (5)

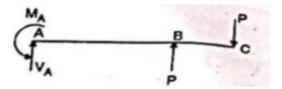


1.(c) Draw the S.F and B.M diagrams for the beam loaded shown in the figure. (5)





 $\therefore M_A = Pa$ $\sum F_y = 0$ $\therefore V_A - P - P = 0$ $\therefore V_A = 0$



Shear force calculations:

- $SF_{CR} = 0$
- $SF_{BR} = P$
- $SF_{BL} = 0$

$$SF_A = 0$$

Bending moment calculations:

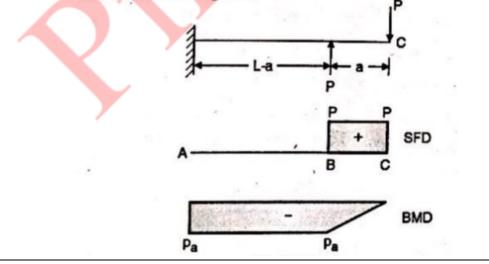
 $BM_C = 0$

 $BM_B = -P X a = -Pa$

$$BM_{AR} = -PL + P(L-a) = -Pa$$

 $BM_{AL} = -Pa + Pa = 0$

S.F and B.M. Diagram :



1.(d) Calculate the bursting pressure for a cold drawn seamless steel tubing of 60 mm inside diameter and 2 mm wall thickness. Ultimate strength of steel is 380 N/mm². (5)



d = 60mm, t= 2mm

$$\sigma_h$$
 = 380 N/mm²
 $\sigma_h = \frac{pd}{2t}$
 $\therefore 380 = \frac{p \times 60}{2X2}$
 $\therefore \mathbf{p} = 25.33 \text{ Mpa}$

1.(e) Find the maximum power that can be transmitted through a 50 mm diamter shaft at 150 rpm, if the maximum permissible shear stress in the shaft is 80 N/mm². (5)

Polar moment of inertia $= \frac{\pi d^4}{32} = 613.59 \times 10^3$ Using torsional formula, $\frac{T}{J} = \frac{\tau}{R}$ \therefore T = 1.963 x 10³ N-m Now, power transmitted, P $= \frac{2\pi NT}{60}$ \therefore P= 30.84 x 10³ watt \therefore P= 30.84 kW

2.(a) A beam weighing 450 N is held horizontal by three vertical wires, one attached to the middle of the beam and the others to the ends of the beam. The outer wires are of brass with 1.25mm diameter and the central wire is of steel with 0.625 mm diameter. Estimate the stresses induced in the wires, assuming that the beam is rigid and the wires are of same length and unstretched before attaching to the beam. Take Young's moduli of brass as $8.6 \times 10^4 \text{ N/mm}^2$ and of steel as $2.1 \times 10^5 \text{ N/mm}^2$. (10)

$$E_{\text{steel}} = 2.1 \text{ x } 10^5 \text{ N/mm}^2$$
$$E_{\text{brass}} = 8.6 \text{ x } 10^4 \text{ N/mm}^2$$
$$\delta l_s = \frac{l_s \text{ x } \sigma_s}{E_s}$$
Similarly, $\delta l_b = \frac{l_b \text{ x } \sigma_b}{E_b}$

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Since increase in the length of steel and brass wire is equal, therefore equating equations,

 $\frac{l_{\rm s} \, {\rm x} \, \sigma_{\rm s}}{2.1 \, {\rm x} \, 10^5} = \frac{l_{\rm b} \, {\rm x} \, \sigma_{\rm b}}{8.6 \, {\rm x} \, 10^4}$

$$\therefore \sigma_s = 2.442 \sigma_b$$

Load supported by the three wires = 450N

$$\therefore 450 = \sigma_{\rm s} A_{\rm s} + \sigma_{\rm b} A_{\rm b}$$

 $\therefore 450 = 1.98 \sigma_b$

 $\therefore \sigma_{b} = 227.27 \text{ MPa}$

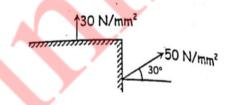
 $\therefore \sigma_s = 555 \text{ MPa}$

2.(b) At a point in a material under stress, the intensity of the resultant stress on certain plane is 50 N/mm² inclined at 30° to the normal of that plane. The stress on a plane at right angles to this has a tensile component of intensity 30 N/mm². Find: (10)

(i) The resultant stress on the second plane

(ii) The principle plane and stresses

(iii) Plane of maximum shear and intensity



From the given data, we first complete the figure indicating the stresses acting at a point.

From figure,

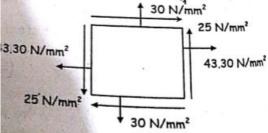
 $\sigma_x = 43.30 \text{ N/mm}^2$

 $\sigma_y = 30 \text{ N/mm2}$

 $\tau_{xy} = 25 \text{ N/mm}^2$

(i)Resultant stresses on second plane





(ii)To find principal stresses P1 and P2 and their directions θ_1 and θ_2

Using P₁, P₂ =
$$\frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}$$

 \therefore Puting the values, we get,

 $P_1 = 62.52 \text{ N/mm}^2$

 $P_2 = 10.78 \text{ N/mm}^2$

Using
$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

 $\therefore \theta = 37.55$

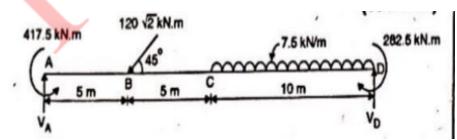
$$\theta_2 = \theta_2 + 90 = 127.55^{\circ}$$

(iii)To find maximum shearing stress and their directions

Using,
$$\tau_{\text{max}} = \pm \frac{P_1 - P_2}{2} = \pm 25.87 \text{N/mm}^2$$

 $\theta_3 = \theta_1 + 45 = 82.55^\circ$
 $\theta_4 = \theta_3 + 90 = 172.55^\circ$

3.(a) For the beam shown draw S.F. and B.M. diagram and mark important points. (10)



Step 1: Support Reactions

 $\sum M = 0$

 $-417.5 + 282.5 + (120\sqrt{2}\sin 45^{\circ}) \ge 5 + (7.5 \ge 10) (5+10) - V_{D} \ge 20 = 0$

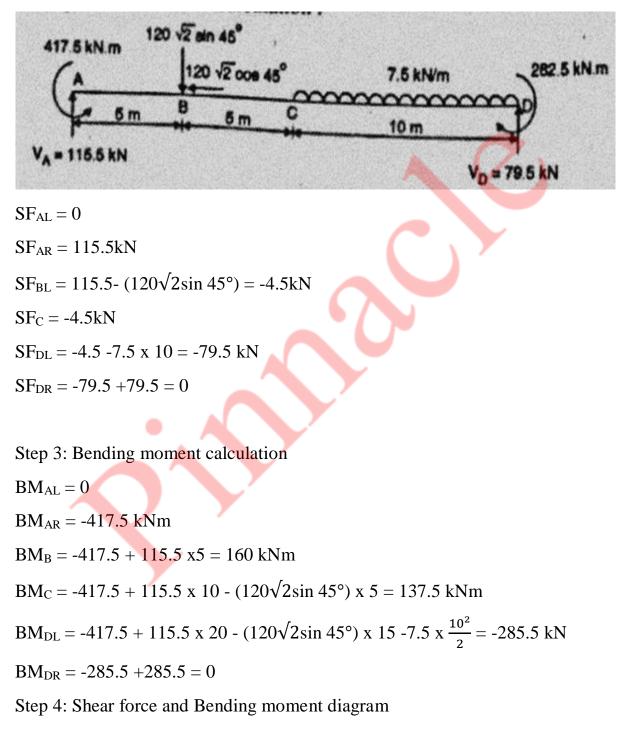


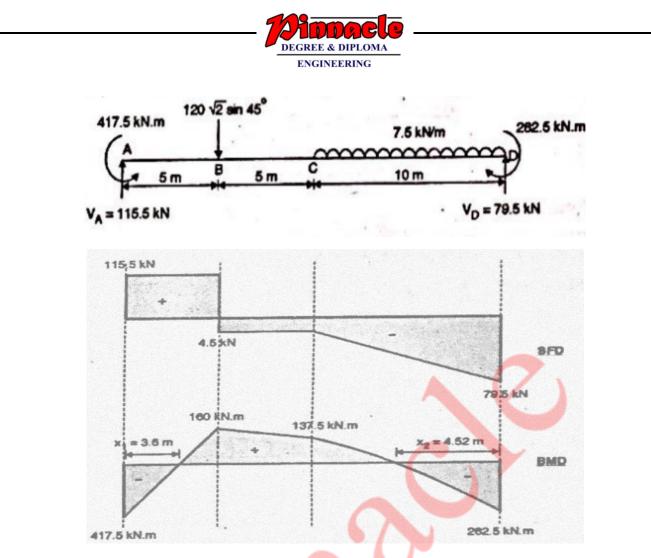
 $\therefore - V_D = 79.5 \text{ kN}$

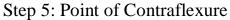
 $\sum F = 0$

 $:: V_A = 115.5 \text{ kN}$

Step 2: Shear force calculations







Consider BMD between portion AB by similarity of triangles,

 $\frac{x_1}{417.5} = \frac{5 - x_1}{160}$

 \therefore x₁ = 3.6 m from point A

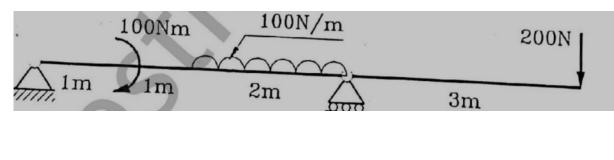
Now consider a section at distance x_2 from pt. D

 $BM_{xx} = 0$

$$-282.5 + 79.5 x_2 - 7.5 \frac{x_2^2}{2} = 0$$

 $\therefore x_2 = 4.52 \text{ m from point } D$

3.(b) Determine the slope and deflection at the free end of the beam loaded as shown. E=200GPa, I= $14 \times 10^6 \text{ m}^4$ (10)



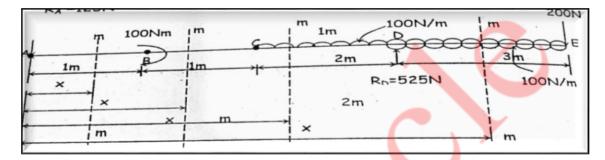
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Let R_A and R_D are the support reaction at A and D.

 $\sum M_A = 0$ -R_D x 4 + 100 + (100 x2) (1+2) + 200x7 = 0 $\therefore R_D = 525 \text{ kN}$ $\sum F_y = 0$ R_A + R_D - 100 x 2 -200 = 0

$$R_A = 125 \text{ kN}$$



$$\operatorname{EI}\frac{d^2y}{dx^2} = -125x|_0^1 + 100(x-1)^0|_0^2 - \frac{100(x-2)^2}{2}|_0^4 + 525(x-4) - \frac{100(x-4)}{2}|_0^7$$

Slope Equation

$$\operatorname{EI}\frac{\mathrm{d}y}{\mathrm{d}x} = \operatorname{C}_1 - \frac{125x^2}{2} \Big|_0^1 + 100(x-1)\Big|_0^2 - \frac{100(x-2)^3}{6}\Big|_0^4 + \frac{525(x-4)^2}{2} - \frac{100(x-4)^3}{6}\Big|_0^7$$

Deflection equation

$$EI.y = C_1 x + C_2 - \frac{125x^3}{6} \Big|_0^1 + \frac{100(x-1)^2}{2} \Big|_0^2 - \frac{100(x-2)^4}{24} \Big|_0^4 + \frac{525(x-4)^3}{6} - \frac{100(x-4)^4}{24} \Big|_0^7$$

At x=0, y=0
$$\therefore$$
 C₂ = 0

At x=4, y=0 \therefore C₁ = -237.5

i)To find deflection at point E, substituting x=7 in deflection

$$EI.y = (-237.5)(7) + 0 - \frac{125(7)^3}{6} + \frac{100(7-1)^2}{2} - \frac{100(7-2)^4}{24} + \frac{525(7-4)^3}{6} - \frac{100(7-4)^4}{24}$$

 \therefore y = -2.709 x 10⁻¹⁵

(ii) To find slope at end A

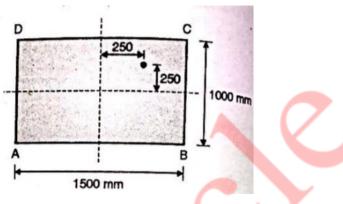
Substituting x = 7 in slope equation.

$$\operatorname{EI}\frac{d^2y}{dx^2} = (-237.5) - \frac{127(7^2)}{2} + 100(7-1) - + \frac{525(7-4)^2}{2} - \frac{100(7-4)^2}{6}$$



$$EI\frac{dy}{dx} = -2870.83$$
$$\therefore \frac{dy}{dx} = 1.025 \text{ x } 10^{-15} \text{ radians}$$

4.(a) A rectangular pier is subjected to a compressive load of 450 kN as shown in the figure. Find the stress intensities at the four corners of the pier. (10)



$$A=b x d = 1 x 1.5 = 1.5 m^2$$

and moment of Inertia of the pier about x-x axis

$$I_{xn} = \frac{bd^3}{12} = 0.125 \text{ m}^4$$

Similarly, $I_v = 0.281 \text{ m}^4$

We also know that moment due to eccentricity or load along x-x axis.

$$M_x = P. e_x = 450 \times 0.25 = 112.5$$

 $M_y = P. e_y = 450 \ge 0.25 = 112.5$

From geometry of the loading, we find that distance between y-y axis and the corners D and C

$$x = \frac{1.5}{2} - 0.75 = 0.75 m$$

Similarly, distance between x-x axis and the corner D and A

$$y = \frac{1}{2} = 0.5 \text{ m}$$

.

From above we can calculate stresses at each corner as follows,

$$\sigma = \sigma_{o} \pm \sigma_{bxx} \pm \sigma_{yy}$$

$$\sigma_{A} = \sigma_{o} - \sigma_{bxx} - \sigma_{yy} = 300 - 450 - 300 = -450 \text{ kN/m}^{2}$$

$$\sigma_{\rm B} = \sigma_{\rm o} - \sigma_{\rm bxx} + \sigma_{\rm yy} = 300 - 450 + 300 = 150 \text{ kN/m}^2$$

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 $\sigma_{\rm C} = \sigma_{\rm o} + \sigma_{\rm bxx} + \sigma_{\rm yy} = 300 + 450 + 300 = 1050 \text{ kN/m}^2$

 $\sigma_{\rm D} = \sigma_{\rm o} + \sigma_{\rm bxx}$ - $\sigma_{\rm yy} = 300 + 450$ - $300 = \textbf{450 kN/m^2}$

4.(b) Internal diameter of a hollow shaft is 0.6 of its external diameter. It has to transmit 300 kW power at 80rpm. If the shear stress is not to exceed 60 N/mm², find the internal and external diameters of the shaft, assuming that the maximum torque is 1.4 times the mean torque. (10)

Hollow shaft with internal diameter (d) and outer diameter(D)

0.6D=d
$$\tau_s = 60 \text{ N/mm}^2$$

 $T_{max} = 1.4 T_{mean}$ P= 300 kW
Now, P= $\frac{2\pi NT}{60}$
P = 300 x 10³ watts N= 80 rpm
Putting these values, we get
 $T_{mean} = 35809.86 \text{ N-m}$
 $\therefore T_{max} = 1.4 \text{ x } 35809.86 = 50133.80 \text{ x } 10^3 \text{ N-mm}$
For hollow shaft torque transmitted is given by

$$\therefore T = \frac{\pi}{16} \cdot \tau_{s} \left(\frac{D^{4} - d^{4}}{D} \right)$$
$$= \frac{\pi}{16} \cdot (60) \left(\frac{D^{4} - (0.6D)^{4}}{D} \right)$$

∴ D = 169.72 mm

∴d= 101.83 mm

5.(a) A 200 kg weight is dropped on to a collar at the lower end of a vertical bar of 3 m long and 28 mm diameter. Calculate the height of drop, if the maximum instantaneous stress is not exceeding 120 N/mm².What is the corresponding instantaneous elongation? Take $E= 2x 10^5$ N/mm². (10)

Ans.

Given: Weight=200 kg = 200 x 9.81 = 91==1962 N



Dig of rod(d) = 28 mm

Length of rod (l)= 3m = 3000 mm

E=2 x 10⁵ N/mm2

 σ instantaneous = 120 N/mm2

Instantaneous stress for rod is given by,

$$\sigma = \frac{P}{A} \left(1 + \sqrt{1 + \frac{2EAh}{PL}} \right)$$

$$120 = \frac{1962}{\frac{\pi}{4}(28)^2} \left(1 + \sqrt{1 + \frac{(2)(2 \times 10^5)\left(\frac{\pi}{4} \times 28^2\right)h}{1962 \times 3000}} \right)$$

$$h=32 \text{ mm}$$
Now, $E = \frac{stress}{strain}$

$$(2 \times 10^5) = \frac{120}{e}$$

$$e = \frac{120}{2 \times 10^5}$$

$$e = 6 \times 10^{-4}$$
Let, instantaneous elongation = (δt)

$$e = \frac{\delta t}{3000} \qquad \delta t = 1.8 \text{ mm}$$
Instantaneous elongation $(\delta t) = 1.8 \text{ mm}$

5.(b) A simply supported beam, with a span of 1.3 m and a rectangular cross section of 1.3 m and a rectangular cross section of 150 mm wide and 250 mm deep, carries a concentrated load of W at center. If the allowable stresses are 7 N/mm2 for bending and 1 N/mm2 for shear, what is the value of the safe load W? (10)

Shear force and bending moment diagrams of given beam is as shown in the figure.

The maximum shear force $=\frac{W}{2}N$



The maximum BM =
$$\frac{WL}{4}Nm$$

 $\sigma_b = 7 N/mm^2$

 $\tau = 1 N / mm^2$

Bending Stress criteria:

Using bending equation,

$$\frac{\sigma_b}{y_{max}} = \frac{M_{max}}{F}$$

$$I = \frac{bd^3}{12} = \frac{150 \times 250^3}{12}$$

$$I = 195.31 \times 10^6 \text{ mm}^4$$

$$y_{max} = \frac{250}{2} = 125mm$$

$$\frac{7}{125} = \frac{W(1.3 \times 10^3)}{\frac{4}{195.31 \times 10^6}}$$

W= 33653.84 N

W=33.65 kN

Shear Stress Criteria:

Maximum shear force in beam = W/2

Cross-section area of beam= $150 \times 250 = 375000 \text{ mm}^2$

Average shear stress = $\frac{shear force}{shear area}$

$$\tau_{avg} = \frac{W/2}{37500} = (1.33 \times 10^{-5})W$$

In a rectangular beam section, the maximum shear stress should not exceed 1 MPa

Therefore, $(2 \times 10-5) \text{ W} = 1$

W = 50000 N

W = 50 kN

Maximum safe load is there for minimum of above two results



W = 33.65 kN

6.(a) A hollow cast iron column of 200 mm external diameter 150 mm internal diameter and a 8 m long has both ends fixed. It is subjected to axial compressive load. Taking factor of safety as 6, $\sigma_c = 560$ N/mm², $\alpha =$ $\frac{1}{1600}$, determine the safe Rankine load. (10)D= 200 mm d= 150mm L = 8m = 800mmColumn is fixed at both ends $L_{e} = \frac{L}{2} = 4000 \text{ mm}$ $\sigma_c = 560 \ N/mm^2$ FOS=6 M.I. = $\frac{\pi}{64}$ (D⁴ – d⁴) = 53.68 x 10⁶ mm⁴ Area = $\frac{\pi}{4}$ (D² – d²) = 13.74 x 10³ mm² $K = \sqrt{\frac{I}{A}} = \sqrt{\frac{53.68 \times 10^6}{13.74 \times 10^3}} = 62.50 \text{ mm}$ $\therefore \text{ Rankine crippling load} = \frac{\sigma_c \cdot A}{1 + \alpha (\frac{Le}{L})^2} = 360.21 \text{ kN}$ ∴ Safe Rankine Load = 360.21 kN

6.(b) A weight pf 200 kN is supported by three adjacent short pillars in a row, ecah 500 mm² in second. The central pillar is made of steel and the outer ones are of copper. The pillars are adjusted such that 15° C each carries equal load. The temperature is then raised to 115° C. Estimate the stresses in each pillar at 15° C and 115° C. (10)

Take $E_s = 2 \ge 10^5 \text{ N/mm}^2 E_C = 0.8 \ge 10^5 \text{ N/mm}^2$ $\alpha_s = 1.2 \ge 10^5 / {}^{0}\text{C}$ $\alpha_c = 1.85 \ge 10^5 / {}^{0}\text{C}$ $P = 200 \text{ kN} = 200 \ge 10^3 \text{ N}$ Now, $\delta l_s = \delta l_c$ $\therefore \frac{\sigma_s .l}{2 \ge 10^5} = \frac{\sigma_c .l}{0.8 \ge 10^5}$ $\therefore \sigma_s = 2.5 \sigma_c$



Also,

 $200 \text{ x } 10^3 = \sigma_s A_s + \sigma_c A_c$

Putting the values of σ_s and $A_s,\,A_c$

We get,

 $\sigma_c = 114.28 \text{ N/mm}^2$

$\therefore \sigma_{s} = 285.7 \text{ N/mm}^{2}$

Stresses before the rise in temp = 285.7 N/mm^2

When temperature is increased,

Force in steel = Force in copper

 $P_s = P_c$

 $\therefore \sigma_{s} A_{s} = \sigma_{c} A_{c}$

Now, $A_s = A_c$

 $\therefore \sigma_{\rm s} = \sigma_{\rm c}$

Now,

$$\frac{\sigma_{\rm s}}{E_{\rm s}} + \frac{\sigma_{\rm c}}{E_{\rm c}} = t (\alpha_{\rm c} - \alpha_{\rm s})$$

Putting the respective values, we get

 $\sigma_{\rm c}(1.75 \text{ x } 10^{-5}) = 6.5 \text{ x } 10^{-4}$

 $\therefore \sigma_c = 37.14 = \sigma_s$